DIGITALNA TEHNIKA I

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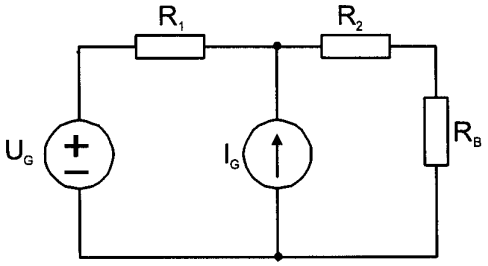
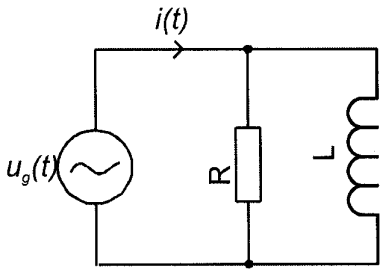
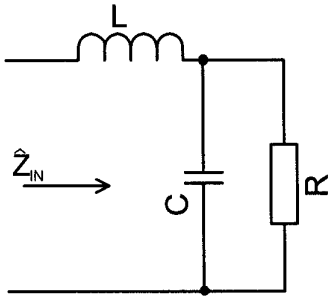
1. Določite velikost bremenskega upora R_B tako da bo prenesena moč iz generatorja na breme maksimalna. Uporabite Theveninovo nadomestno vezje in izrek o maksimalni preneseni moči. Upoštevajte: $U_G=5V$ $I_g=5A$ $R_1=2\Omega$ $R_2=3\Omega$



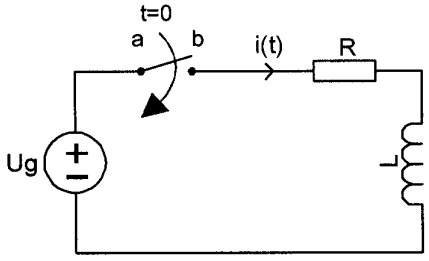
2. Vezje na sliki je vzbujano z izmenično napetostjo $u_g(t)=5V\ cos(100t)$. Določite potek toka $i(t)$. Upoštevajte: $L= 0.3H$, $R= 250\Omega$



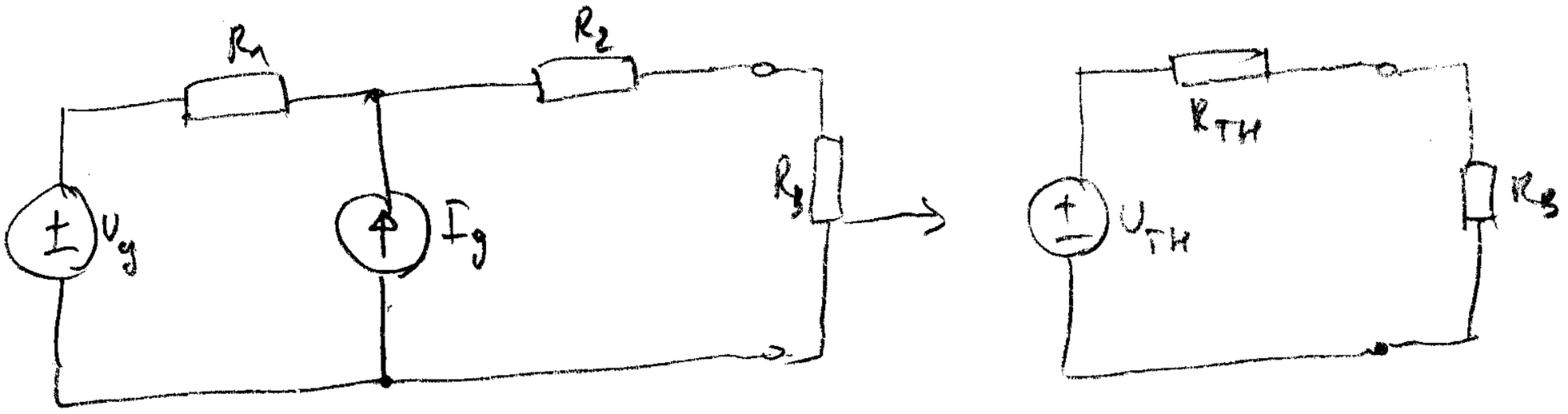
3. Določite serijsko resonančno frekvenco vezja na sliki.
Upoštevajte: $L=10\mu H$ $R=10M\Omega$ $C=1000\mu F$



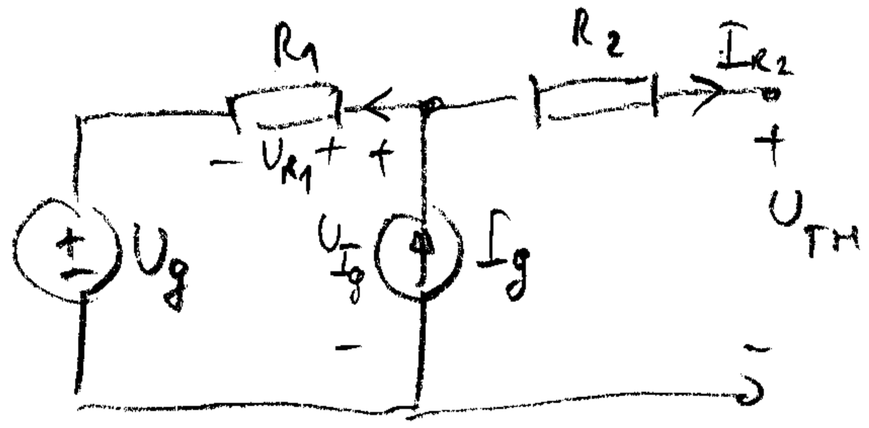
4. V trenutku $t=0$ sklenemo stikalo. Izračunajte tok $i(t)$ skozi induktivnost L z uporabo Laplaceove transformacije. Začetni tok $i(t)$ skozi induktivnost je $i(0^+)=0A$!
Upoštevajte: $L=1H$ $R=1\Omega$ $U_g=5V$



Čas pisanja je 60min. Vsaka naloga je vredna 10 točk. Na list z rešitvami se podpišite in napišite še vpisno številko ter kateri predmet pišete. Rezultati bodo objavljeni na https://estudent.fri.uni-lj.si

① 

U_{TH}:

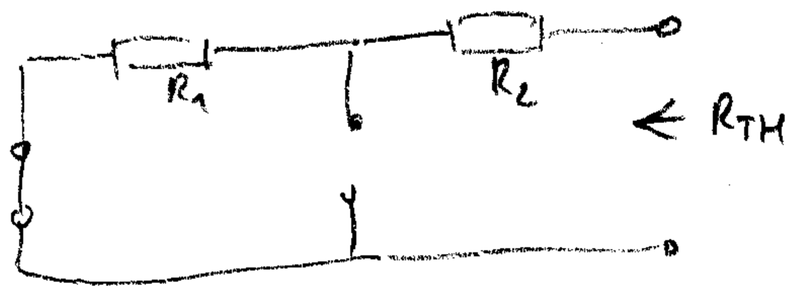


$I_{R_2} = 0 \rightarrow U_{TH} = U_{I_g}$

$U_{I_g} = U_g + U_{R_1} = U_g + R_1 \cdot I_g = 5 + 2 \cdot 5 = \underline{15V}$

$U_{TH} = 15V$

R_{TH} :



$R_{TH} = R_1 + R_2 = 5\Omega$

$\boxed{P_{MAX} \rightarrow R_{TH} = R_B}$ $\quad R_B = 5\Omega$

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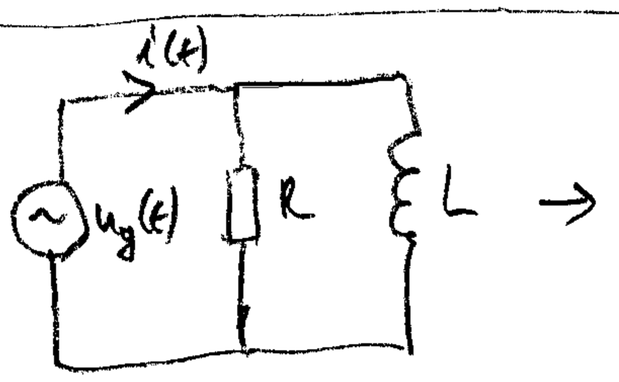
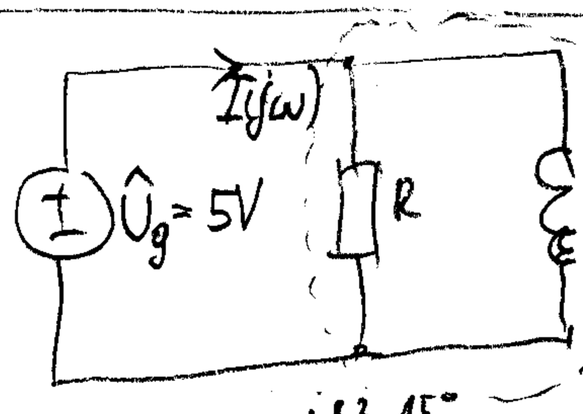
② 



$\hat{Z}_L = j\omega L = j \cdot 100 \cdot 0{,}3 = j30\Omega$

$\Rightarrow \hat{Z}_{Sk} = \frac{R \cdot \hat{Z}_L}{R + \hat{Z}_L} = \frac{250 \cdot j30}{250 + j30} = \frac{7500 e^{j90°}}{251{,}8 e^{j6{,}8°}}$

$\hat{Z}_{Sk} = 29{,}8 \cdot e^{j83{,}15°}\Omega$

$\hat{I} = \frac{\hat{U}_g}{\hat{Z}_{Sk}} = \frac{5V}{29{,}8 e^{j83{,}15°}} = 0{,}168 e^{-j83{,}15°} A$

$\boxed{i(t) = 0{,}168 \cos(100t - 83{,}15°) A}$

③

$$\hat{Z}_{IN} = j\omega L + \frac{1}{j\omega C} \| R$$

$$\hat{Z}_{IN} = j\omega L + \frac{\left(\frac{1}{j\omega C} \cdot R\right)\left(R - \frac{1}{j\omega C}\right)}{\left(R + \frac{1}{j\omega C}\right)\left(R - \frac{1}{j\omega C}\right)} = j\omega L + \frac{\frac{1}{j\omega C} \cdot R\left(R - \frac{1}{j\omega C}\right)}{R^2 + \frac{1}{\omega^2 C^2}}$$

$$\hat{Z}_{IN} = \frac{j\omega L\left(R^2 + \frac{1}{\omega^2 C^2}\right) + \frac{1}{j\omega C} R\left(R - \frac{1}{j\omega C}\right)}{R^2 + \frac{1}{\omega^2 C^2}}$$

Zanima nas samo imaginarni del impedance Z

$$\hat{Z}_{IN} = \frac{j\omega L\left(R^2 + \frac{1}{\omega^2 C^2}\right) - j\frac{R^2}{\omega C} + \frac{R}{\omega^2 C^2}}{R^2 + \frac{1}{\omega^2 C^2}} = Re[\hat{Z}_{IN}] + j\,Im[\hat{Z}_{IN}]$$

Kriterij za serijsko resonanco:

$$Im(\hat{Z}_{IN}) = 0 = \omega L\left(R^2 + \frac{1}{\omega^2 C^2}\right) - \frac{R^2}{\omega C}$$

$$\frac{R^2}{\omega C} = \omega L\left(R^2 + \frac{1}{\omega^2 C^2}\right)$$

$$\frac{R^2}{\omega^2 LC} = R^2 + \frac{1}{\omega^2 C^2} \quad / : R^2$$

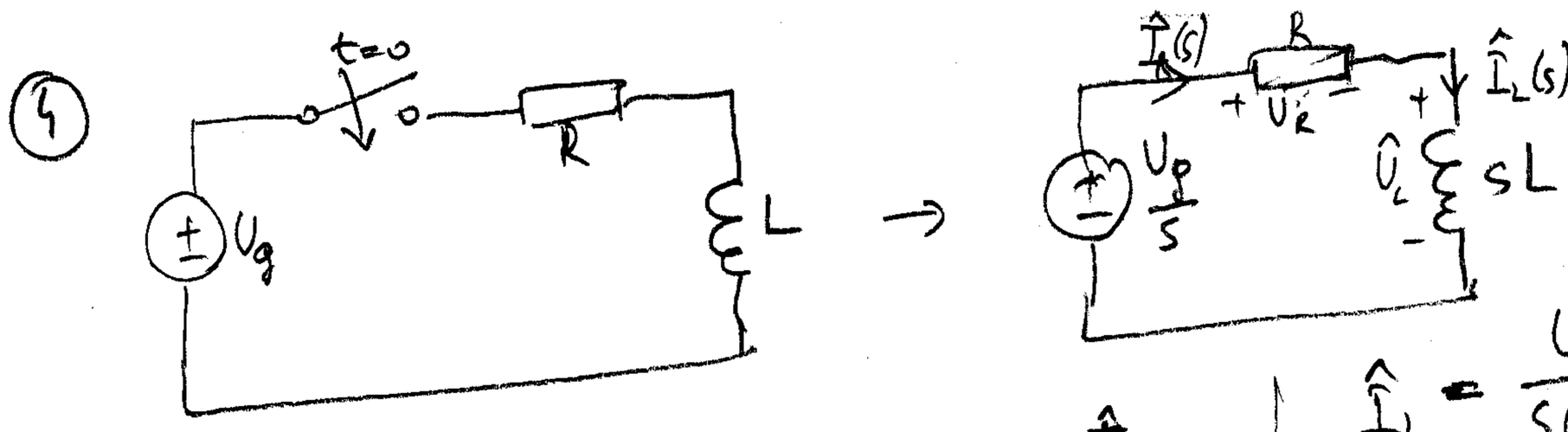
$$\frac{1}{\omega^2 LC} = 1 + \frac{1}{\omega^2 R^2 C^2}$$

$$\frac{1}{\omega^2}\left[\frac{1}{LC} - \frac{1}{R^2 C^2}\right] = 1$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{1}{R^2 C^2}} = \sqrt{\frac{1}{10 \cdot 10^{-6}\,\frac{Vs}{A} \cdot 1000 \cdot 10^{-6}\,\frac{As}{V}} - \frac{1}{(10 \cdot 10^6)^2\,\frac{V^2}{A^2}\,(1000 \cdot 10^{-6})^2\,\frac{A^2 s^2}{V^2}}}$$

$$\omega = \sqrt{\underbrace{\frac{1}{10^3 \cdot 10^{-12}}}_{10^9} - \frac{1}{10^8}} \approx \sqrt{10^9} = 31{,}622\,rd/s \rightarrow f = \frac{\omega}{2\pi} \doteq \underline{\underline{5\,kHz}}$$

(4)



KNZ: $\frac{U_g}{s} = U_R + U_L$

KRZ: $\hat{I}_R = \hat{I}_L$

$\hat{U}_L = sL \cdot \hat{I}_L$

$\hat{U}_R = R \cdot \hat{I}_R$

$$\frac{U_g}{s} = R \cdot \hat{I}_L + sL\,\hat{I}_L = (R+sL)\,\hat{I}_L \rightarrow$$

$$\hat{I}_L = \frac{U_g}{s(R+sL)} = \frac{5V}{s(s+1)}$$

$$\hat{I}_L = \frac{A}{s} + \frac{B}{s+1} = \frac{5}{s} - \frac{5}{s+1}$$

$$A = \lim_{s\to 0} \frac{5V}{s+1} = 5$$

$$B = \lim_{s\to -1} \frac{5V(s+1)}{(s+1)s} = -5$$

$$i_L(t) = 5 \cdot 1(t) - 5e^{-t} \cdot 1(t)$$