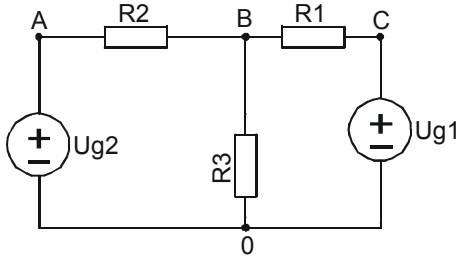
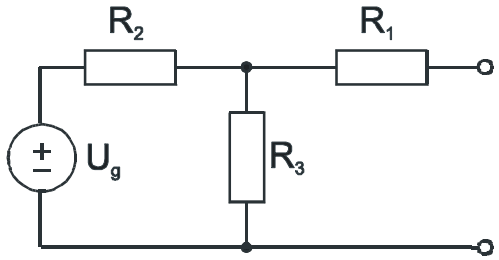
DIGITALNA TEHNIKA I

1.kolokvij 15. 12. 2004

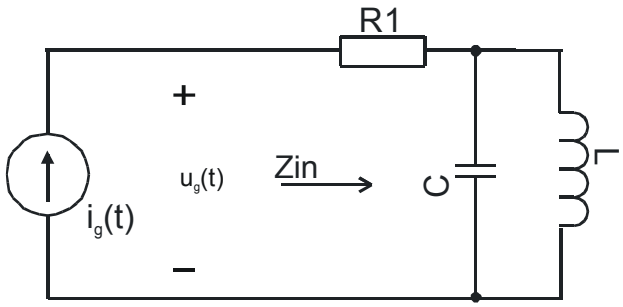
1. Izračunajte napetosti v vozliščih *A, B* in *C*.
   Upoštevajte: $Ug_1=5V$ $Ug_2=1V$ $R_1=1\Omega$ $R_2=2\Omega$ $R_2=3\Omega$



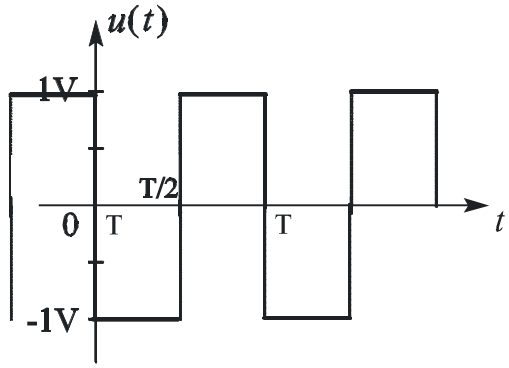
2. Izračunajte parametre Nortonovega nadomestnega vezja za spodnji neidealni generator.
   Upoštevajte: $Ug=5V$ $R_1=1\Omega$ $R_2=2\Omega$ $R_2=3\Omega$



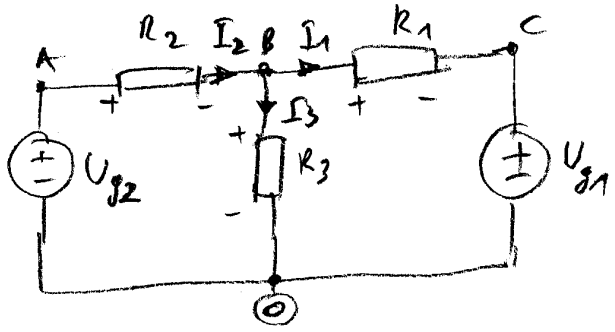
3. Izračunajte vhodno impedanco $Z_{in}(\omega)$ vezja na sliki in napetost $u_g(t)$.
   Upoštevajte $L= 1H$ $C=1F$ $R_1=1\Omega$ $i_g(t)=1\cos(\omega t)\ A$ $\omega=2rd/s$



4. Določite koeficiente Fourierove vrste za signal *u(t)* na spodnji sliki in zapišite njene prve štiri člene.



Čas pisanja je 60min. Vsaka naloga je vredna 10 točk. Na list z rešitvami se podpišite in napišite še vpisno številko ter kateri predmet pišete. Rezultati bodo objavljeni na http://dt.fe.uni-lj.si/



$U_{g2} = U_A$

$U_C = U_{g2}$

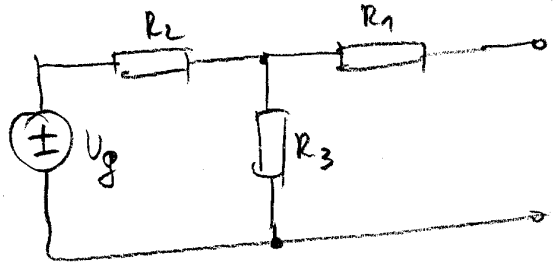
KTZ (B)

$I_2 = I_1 + I_3$

$$\frac{U_A - U_B}{R_2} = \frac{U_B - U_C}{R_1} + \frac{U_B}{R_3}$$

$$\frac{U_A}{R_2} + \frac{U_C}{R_1} = \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] U_B$$

$$U_B = \frac{\frac{5V}{2\Omega} + \frac{1V}{1\Omega}}{\frac{1}{1\Omega} + \frac{1}{2\Omega} + \frac{1}{3\Omega}} = \underline{1{,}91\ V}$$



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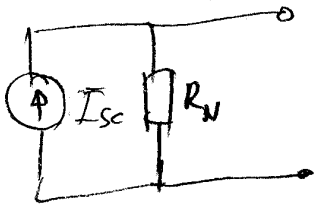
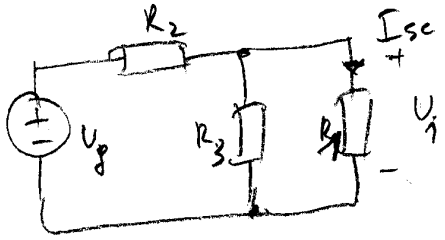


I_{sc}



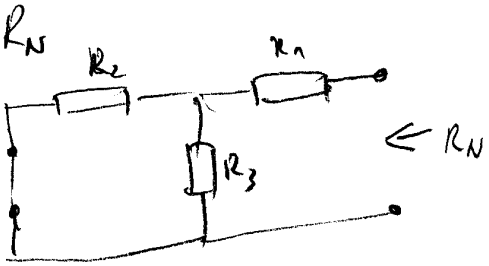
$$R_1 \| R_3 = \frac{R_1 R_3}{R_1 + R_3} = \frac{1 \cdot 3}{1+3} = \frac{3}{4}$$

$$U_1 = \frac{R_1 \| R_3}{R_2 + (R_1 \| R_3)} U_g$$

$$U_1 = \frac{\frac{3}{4}\Omega}{2\Omega + \frac{3}{4}\Omega} \cdot 5V = \underline{1{,}364\ V}$$

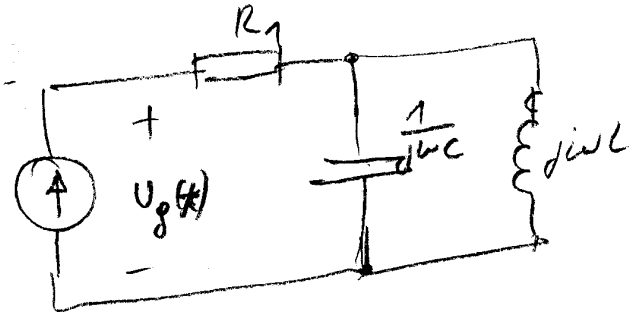
↳ $I_{sc} = I_{R_1} = \frac{U_1}{R_1} = \underline{1{,}364\ A}$

R_N



$$R_N = R_1 + (R_2 \| R_3) \quad ; \quad R_2 \| R_3 = \frac{R_2 R_3}{R_2 + R_3} = \frac{2 \cdot 3}{2+3}$$

$$R_N = 1\Omega + \frac{6}{5}\Omega = \frac{11}{5}\Omega = \underline{2{,}2\Omega}$$



$$\hat{Z}_{IN} = R_1 + \left(\frac{1}{j\omega C} \| j\omega L\right)$$

$$\frac{1}{j\omega C} \| j\omega L = \frac{\frac{1}{j\omega C} \cdot j\omega L}{\frac{1}{j\omega C} + j\omega L} = \frac{1}{j\omega + \frac{1}{j\omega}}$$

$$\hat{Z}_{IN}(j\omega) = R_1 + \frac{1}{j\left(\omega - \frac{1}{\omega}\right)} = \underline{1 - j\frac{1}{\omega - \frac{1}{\omega}}} \quad ; \quad \hat{Z}_{IN}(j\omega) = 1 - j\frac{1}{2 - \frac{1}{2}}$$

$$Z_{in}(j\omega) = \underline{1 - j\frac{2}{3}}$$

$$U_g(\omega t) = \hat{I}_g(\omega t) \cdot \hat{Z}_{IN}(j\omega) = 1A\, e^{-j0^\circ} \cdot 1{,}20 \cdot e^{-j33{,}69^\circ} = 1{,}20\, e^{-j33{,}69^\circ}$$

$$u_g(t) = 1{,}20V \cdot \cos(\omega t - 33{,}69^\circ)$$

Signal je lih $f(x) = -f(-x)$; $a_n = 0$; $\omega = \frac{2\pi}{T}$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)\,dt$$

$$b_n = \frac{2}{T}\int_0^{T/2} 1V \cdot \sin(n\omega t)\,dt + \frac{2}{T}\int_{T/2}^{T} (-1V)\sin(n\omega t)\,dt$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

$$\frac{2}{T}\left(-\frac{1}{n\omega}\right)\cos(n\omega t)\Big|_0^{T/2} + \frac{2}{T}\left(+\frac{1}{n\omega}\right)\cos(n\omega t)\Big|_{T/2}^{T} \qquad \frac{2}{T}\cdot\frac{1}{n\cdot\frac{2\pi}{T}} = \frac{1}{n\pi}$$

$$-\frac{1}{n\pi}\left[\cos\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right) - 1\right] + \frac{1}{n\pi}\left[\cos\left(n\frac{2\pi}{T}\cdot T\right) - \cos\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)\right]$$

$$-\frac{1}{n\pi}\cos(n\pi) + \frac{1}{n\pi} + \frac{1}{n\pi}\cos(n\cdot 2\pi) - \frac{1}{n\pi}\cos(n\pi) \quad ; \quad \cos(n2\pi) = 1 \quad \cos(n\pi) = (-1)^n$$

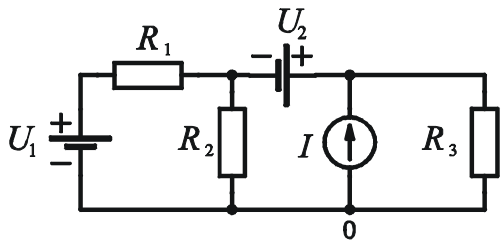
$$b_n = \frac{2}{n\pi} - 2\frac{(-1)^n}{n\pi} = \begin{cases} 0 & ; n = sod \\ \frac{4}{n\pi} & ; n = lih \end{cases}$$

$$u(t) = \frac{4}{\pi}\sin(\omega t) + \frac{4}{3\pi}\sin(3\omega t) + \frac{4}{5\pi}\sin(5\omega t) + \frac{4}{7\pi}\sin(7\omega t) + \ldots$$

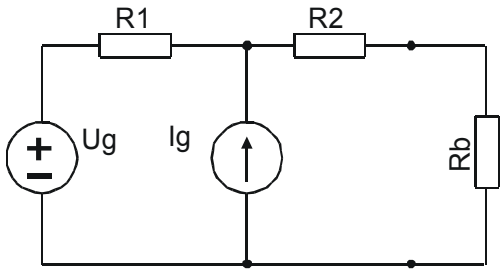
DIGITALNA TEHNIKA (1.del)

1.kolokvij 15. 12. 2004

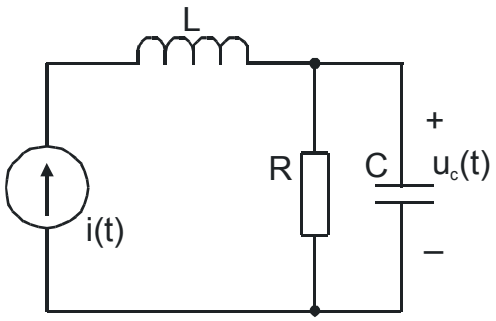
1. Izračunajte napetosti v vozliščih proti referenčnemu vozlišču 0. Vrednosti elementov vezja so: R_1=1Ω R_2=5Ω R_3=3Ω I=5A U_1=4V U_2=7V



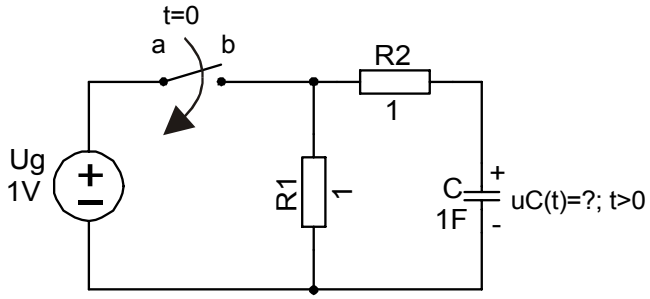
2. Določite Theveninovo nadomestno vezje in velikost bremenskega upora Rb tako, da bo moč na bremenu maksimalna.
   Upoštevajte: Ug=5V Ig=1A R_1=5Ω R_2=4Ω



3. Izračunajte napetost na kondenzatorju $u_C(t)$, če vezje napajamo z generatorjem $i(t)= 4\ sin(\omega t+60^o)$ A.
   Ostali elementi vezja so: $C=1F$ $L=1H$ $R=1\Omega$ $\omega=1\ rd/s$



4. V trenutku $t=0$ sklenemo stikalo. Izračunajte napetost $u_C(t)$ na kondenzatorju C z uporabo Laplaceove transformacije. Začetna napetost $u_C(t)$ na kondenzatorju je $u_C(0^+)$=0V!



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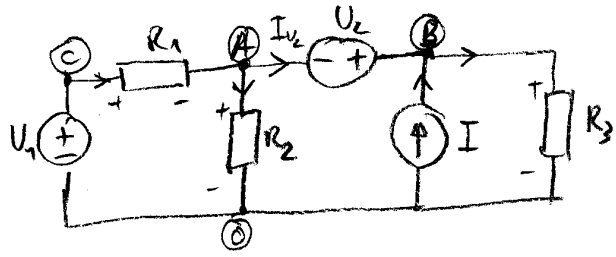

① $U_B - U_A = U_2$

② $U_C = U_1$

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③ $\frac{U_C - U_A}{R_1} = \frac{U_A}{R_2} + I_{U_2}$

④ $I_{U_2} + I = \frac{U_B}{R_3}$

Iz ③ in ④ eliminiramo I_{U_2} in upoštevamo ②

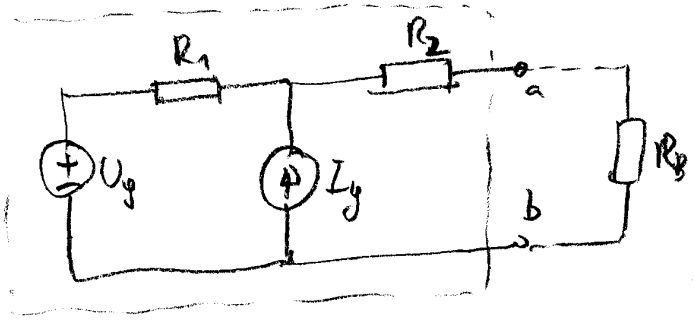
⑤ $\frac{U_1 - U_A}{R_1} = \frac{U_A}{R_2} + \frac{U_B}{R_3} - I$

Vstavimo ① v ⑤

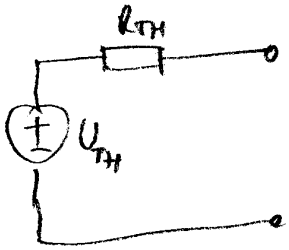
$$\frac{U_1 - U_A}{R_1} = \frac{U_A}{R_2} + \frac{U_2 + U_A}{R_3} - I$$

$$\frac{U_1}{R_1} - \frac{U_2}{R_3} + I = U_A \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

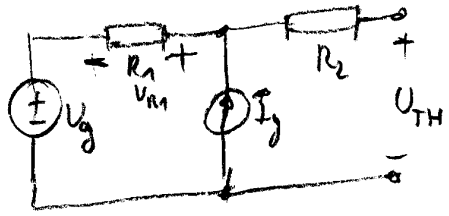
$$\frac{\frac{4V}{1\Omega} - \frac{7V}{3\Omega} + 5A}{\frac{1}{1\Omega} + \frac{1}{5\Omega} + \frac{1}{3\Omega}} = U_A = 4{,}35V \qquad U_B = U_2 + U_A = 11{,}35V$$



Theveninov ekvivalent



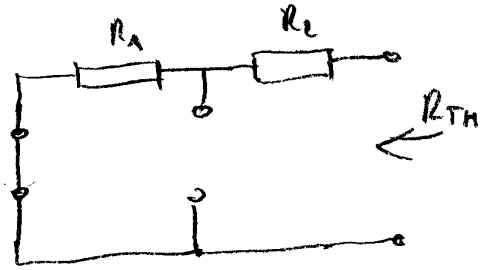
Določanje U_{TH}: med a in b ne priključimo nič (prosti tek)



$$U_{TH} = U_g + U_{R_1} = U_g + I_g \cdot R_1 = 5 + 1.5 = \underline{10V}$$

$$U_{R_1} = I_g \cdot R_1$$

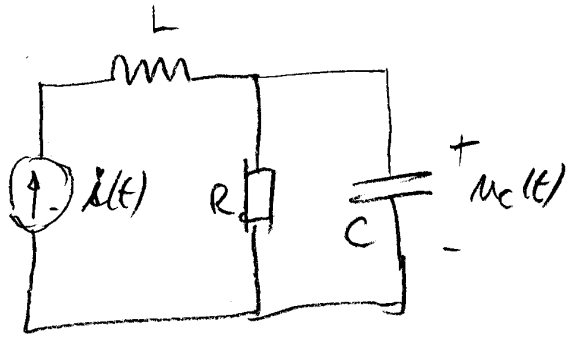
Določimo R_{TH}: Generatorje nadomestimo z 0 vrednostmi.



$$R_{TH} = R_1 + R_2 = 9\Omega$$

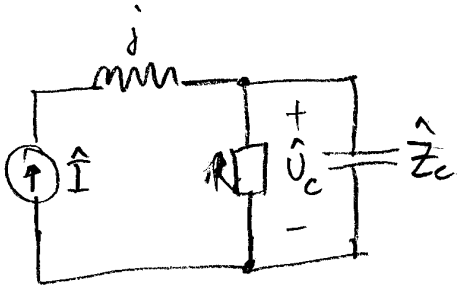
Po izreku o maksimalnem prenosu moči velja $R_B = R_{TH}$; ko je moč na bremenu max. Torej

$$R_B = R_{TH} = \underline{9\Omega}$$



Transformirano vezje:

Napajanje $i(t) = 4\sin(\omega t + 60°)\,A = 4\cos(\omega t - 30°) \rightarrow \hat{I} = 4e^{-j30°}$



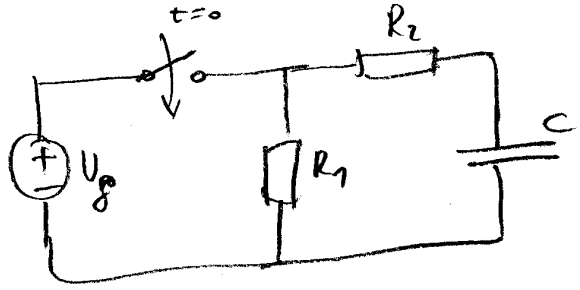
$$\hat{U}_c = \hat{I} \cdot (R \parallel \hat{Z}_c) \; ; \; \hat{Z}_c = \frac{1}{j\omega C}$$

$$R \parallel \hat{Z}_c = \frac{R\hat{Z}_c}{R + \hat{Z}_c} = \frac{1 \cdot \frac{1}{j}}{1 + \frac{1}{j}} = \frac{-j}{1-j}$$

$$R \parallel \hat{Z}_c = \frac{-j(1+j)}{(1-j)(1+j)} = \frac{1-j}{2} = \frac{1}{\sqrt{2}} e^{-j45°}$$

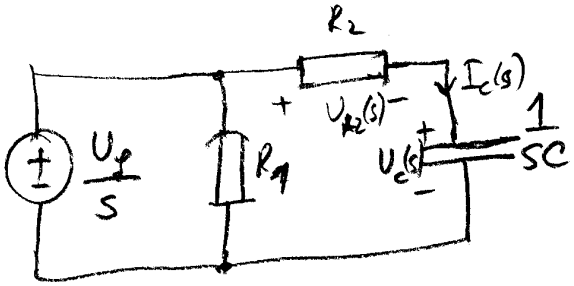
$$\hat{U}_c = 4 \cdot e^{-j30°} \cdot \frac{1}{\sqrt{2}} e^{-j45°} = \frac{4}{\sqrt{2}} e^{-j75°}$$

$$\boxed{u_c(t) = \frac{4}{\sqrt{2}} \cos(t - 75°)}$$



$u_c(0) = 0$

Transformiramo vezje



① $\frac{U_g}{s} = U_{R_2} + U_c(s)$

$U_{R_2} = I_c(s) \cdot R_2$

$U_c(s) = I_c(s) \cdot \frac{1}{sC}$

② $U_{R_2}(s) = R_2 \cdot U_c(s) \cdot sC$

Vstavimo ① v ②

Dobimo

$$\frac{U_g}{s} = R_2 sC\, U_c + U_c(s) = (1 + R_2 C \cdot s) U_c$$

③ $$\frac{U_g}{s(1+R_2Cs)} = V_c$$

Parcialni ulomki:

$$\frac{1}{s(1+R_2Cs)} = \frac{A}{s} + \frac{B}{1+R_2Cs} = \frac{1}{s} - \frac{R_2C}{1+R_2Cs} = \frac{1}{s} - \frac{R_2C}{R_2C(s+\frac{1}{R_2C})}$$

$$A = \lim_{s\to 0} \frac{1}{1+R_2Cs} = 1 \quad ; \quad B = \lim_{s\to -\frac{1}{R_2C}} \frac{1}{s} = -R_2C$$

Inverzna $\mathcal{L}$ transformacija:

$$\mathcal{L}^{-1}\left[\frac{U_g}{s} - \frac{U_g}{s+\frac{1}{R_2C}}\right] = u_c(t) \rightarrow u_c(t) = U_g\left[1 - e^{-\frac{t}{R_2C}}\right] \cdot 1(t)$$